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Innovative Applications of O.R.

Optimal firm growth under the threat of entry[☆]

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ABSTRACT

The paper studies the incumbent-entrant problem in a fully dynamic setting. We find that under an open-loop information structure the incumbent anticipates entry by overinvesting, whereas in the Markov perfect equilibrium the incumbent slightly underinvests in the period before the entry. The entry cost level where entry accommodation passes into entry deterrence is lower in the Markov perfect equilibrium. Further we find that the incumbent's capital stock level needed to deter entry is hump shaped as a function of the entry time, whereas the corresponding entry cost, where the entrant is indifferent between entry and non-entry, is U-shaped.

1. Introduction

The paper studies the incumbent-entrant problem in a fully dynamic setting. Initially the incumbent offers a homogenous product. To increase production capacity the firm can invest to enlarge its capital stock. From some given future point in time on, another firm can enter this market. In case entry takes place, a duopoly market arises with homogenous products. The question is how the incumbent invests in order to anticipate the future entry threat. Basically, it can choose between a policy of entry deterrence and entry accommodation. In the latter case we also investigate what happens after the second firm has entered. Then two firms are in the market and both can invest to increase production capacity.

First, we consider a situation where at some given future point in time an inevitable entry takes place. This allows us to establish the optimal entry accommodation policy.

However, under a Markov perfect equilibrium information structure the incumbent slightly underinvests in the period before entry takes place. The reason is that in such a framework a higher market share is less persistent, because investment rates are directly influenced by capital stocks of the own firm and of the competitor. The entrant just has to increase its own capital stock in order to reduce investments of the incumbent. A second reason for anticipatory underinvestment by the incumbent is that profits are lower in a Markov perfect equilibrium. This reduces the incentive to invest in this market.

Second, we study a framework where market entry requires incurring a fixed entry cost. This enables the incumbent to establish the critical capital stock level it needs to build up in order to deter entry. Entry deterrence is optimal if it is not too costly to build up this level. We establish that for low entry cost entry accommodation will result, for intermediate levels of the entry cost the incumbent will deter entry, and higher entry cost levels imply that the incumbent is a natural monopoly.

The paper essentially contributes to two streams of the literature. The first stream considers an incumbent-entrant framework where the incumbent has the choice between deterring and accommodating entry. The first contributions are Dixit (1979, 1980), Spence (1977), being surveyed in Tirole (1988, Chap. 8). Maskin (1999) extends this literature by adding uncertainty and obtains that the incumbent should hold a higher capacity to deter the entrant. Abbring and Campbell (2007) construct a discrete time model and find that it may happen that incumbents will serve the total market if entry barriers exist for new entrants. Fudenberg and Tirole (1983, 1986) employ a continuous-time model to find that in a Markov perfect equilibrium it is possible that a firm that has a head start in industry

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can deter entry (or at least mobility) by overinvestment. In their model the firms have linear investment costs. They further assume that each firm has an upper bound on the amount of investment and argue that this "serves as a proxy for the more realistic case of convex costs of investment" (Fudenberg & Tirole, 1983, p. 230). Our paper in fact considers this convex cost of investment case, which enables us to explicitly consider how firms accumulate capital over time.

The second stream is the literature on duopoly differential games with the emphasis on capital accumulation. Early contributions include Driskill and McCafferty (1989), Reynolds (1987, 1991), with an overview being provided by Dockner, Jø rgensen, Long, and Sorger (2000). Jun and Vives (2004) compare steady states of open-loop and Markov perfect equilibria, where a full characterization is provided in the linear-quadratic case. We do the same, but where Jun and Vives concentrate on a symmetric game, we consider the incumbent-entrant framework.

This paper extends the static literature on entry deterrence to a dynamic framework. This has also been done in Huisman and Kort (in press) but there firms were allowed to invest only once during a time period of infinite length. In our setting firms are allowed to invest whenever they want, resulting in various incremental changes of the capital stock. We find that in an open-loop information structure the incumbent anticipates entry by overinvesting, whereas the incumbent slightly underinvests in a Markov-perfect equilibrium. Furthermore, a policy of entry deterrence is more worthwhile to pursue in the open-loop framework.

The paper is organized as follows. Section 2 presents the model where entry time is fixed and the entrant can enter the market for free. In Section 3 fixed entry costs are added and the entrant may enter from some given time in the future onwards. Section 4 concludes.

2. The model with fixed entry time

Consider an incumbent-entrant model, where, before (eventual) entry takes place, the market consists of one monopolistic firm, being the incumbent (firm 1). The firm that considers entry is denoted by firm 2. This section considers a scenario where the entry time, T, is exogenously given and known, and entry costs are negligible. This implies that we take firm 2's entry at time T for granted, and our aim is to analyze the effect of firm 2's entry on firm 1's investment behavior prior to entry time T. Section 3 deals additionally with positive entry costs and the timing of entry, provided it takes place at all (positive entry costs may result in a profitable policy of entry deterrence by firm 1).

Firm 1's corresponding model builds on the classical capital accumulation models (see, among many others, Barucci, 1998; Eisner & Strotz, 1963). The capital stock, $K_1(t)$, can be increased by capital investments $I_1(t)$, and decreases with a non-negative depreciation rate $\delta > 0$:

$$\dot{K}_1(t) = I_1(t) - \delta K_1(t), \quad K_1(0) = K_{10}, \tag{1}$$

where K_{10} denotes the initial capital stock at $t = 0$. From now on we assume that $K_{10} = 0$.[2] The capital stock $K_1(t)$ is used to produce output with a linear production function, i.e. $F(K_1(t)) = aK_1(t)$ (without loss of generality we chose $a = 1$). The price of output is determined by an inverse demand function, i.e.

$$p(t) = A - K_1(t), \tag{2}$$

with A being a positive constant. Firm 1's revenue therefore equals

$$R_1(t) = p(t)K_1(t) = (A - K_1(t))K_1(t). \tag{3}$$

The cost of investment consists of linear acquisition costs, $bI_1(t)$, and quadratic costs of implementation, $\frac{c}{2}I_1^2(t)$, where b and c are positive constants.

At entry time T the market switches to competition. The present value of the incumbent's profits earned from there on depends on its capital stock at the switching time $K_1(T)$, and, since the initial capital stock of the entrant equals zero,[3] we can denote these profits by $S(K_1(T))$. This results in the following optimization problem for the incumbent:

$$\max_{I_1(t)} \int_0^T e^{-rt}\left((A - K_1(t))K_1(t) - bI_1(t) - \frac{c}{2}I_1^2(t)\right)dt + e^{-rT}S(K_1(T))$$
$$\text{s.t. } \dot{K}_1(t) = I_1(t) - \delta K_1(t), \quad K_1(0) = 0, \tag{4}$$

where r is the discount rate.

From time T on, two firms compete in an oligopolistic market with homogenous goods. Consequently, the output price after firm 2 has entered, equals

$$p(t) = A - K_1(t) - K_2(t) \tag{5}$$

for both firms. The firms are both profit maximizers, where the time horizon is infinite. Putting things together we arrive at a classical capital accumulation game as presented in Reynolds (1987),[4] i.e.

$$\begin{aligned}
&\text{Firm } 1: \max_{I_1(t)} \\
&\quad \int_T^\infty e^{-rt}\left((A - K_1(t) - K_2(t))K_1(t) - bI_1(t) - \frac{c}{2}I_1^2(t)\right) dt,\\
&\text{Firm } 2: \max_{I_2(t)} \\
&\quad \int_T^\infty e^{-rt}\left((A - K_1(t) - K_2(t))K_2(t) - bI_2(t) - \frac{c}{2}I_2^2(t)\right) dt,\\
&\text{s.t.} \quad \dot{K}_1(t) = I_1(t) - \delta K_1(t),\\
&\qquad \dot{K}_2(t) = I_2(t) - \delta K_2(t), \quad K_2(T) = 0.
\end{aligned} \tag{6}$$

In the same paper this differential game is solved for the open-loop and feedback (or Markov perfect) case. Therefore, we will not repeat the analysis, but only some highlights and key results we need for our economic analysis (see Appendices A–C). Due to the linear quadratic structure it is possible to obtain an analytical solution. This is presented in the following sections for both (open-loop and Markov perfect equilibrium) scenarios.

2.1. Analysis and economic interpretation (Markov perfect equilibrium case)

In the remainder of the paper the superscripts M, MP, and O denote variables that correspond to monopoly (M), or Markov perfect (MP) and open-loop commitment structure (O).

This section deals with a Markov perfect equilibrium structure in the duopoly game that arises after firm 2 has entered. As demonstrated in Reynolds (1987), the Hamilton–Jacobi–Bellman function has 6 solutions, where one is asymptotically stable (for details we refer to Reynolds (1987)). Comparing the steady state solutions reveals that the capital stock as well as the investments of the monopolist always exceed that of a duopoly firm (Markov perfect equilibrium), i.e.

$$\hat{K}_1^M = \frac{A - b(r+\delta)}{2 + \delta c(r+\delta)} > \frac{A - b(r+\delta)}{3 + \delta c(r+\delta) - \frac{\sigma}{\pi - c(r+\delta)}} = \hat{K}_1^{\text{MP}},$$

[2] The analysis of the model with positive K_{10} is completely analogous. Note that this choice is no restriction to the model. Due to the Bellman principle the behavior of the incumbent before time T with positive K_{10} (i.e. $K_{10} = \xi > 0$, entry time T) is completely the same compared to the situation in which the incumbent owns that capital stock at some time $\bar{t}$ with entry time horizon $T + \bar{t}$.

[3] Note that the analysis of this paper is also possible for a positive initial capital stock of the competitor. However, it is more reasonable to assume within this model that the capital stock has to be built up after the entrance.

[4] For other contributions we refer to Dragone, Lambertini, and Palestini (2010), Fershtman and Muller (1984), Reynolds (1991). A capital accumulation game with a capital stock with vintage structure has been dealt with in Wrzaczek and Kort (2012). For a profound overview on dynamic capital accumulation games we refer to Dockner et al. (2000), Long (2010).

where $\sigma < 0$ and $\pi < 0$ are parameters of the value function (see Reynolds, 1987 for details). The superscript MP refers to values from the competition period with the Markov perfect equilibrium structure. Optimal investments are given by

$$I_i = \frac{\beta + \pi K_i + \sigma K_j - b}{c}, \tag{7}$$

where $\beta > 0$ is another parameter of the value function. We thus obtain that firm *i*'s investments depend negatively on the own and on the competitor's capital stock. The reason for the first result is that profit is concave in K_i, i.e. marginal revenue is decreasing. Furthermore, investment depends negatively on the competitor's capital stock, since this stock has a negative effect on the output price, thereby reducing the incentive to invest in this market.

By applying the result of the Markov perfect equilibrium (see Reynolds, 1987) we obtain the following concave salvage value function:

$$S(K_1) = \alpha + \beta K_1 + \frac{\pi}{2}K_1^2,$$

where $\alpha > 0$. The problem of the incumbent before entry takes place has not been analyzed so far in a continuous dynamic framework. As we assume within this section that the entry time T is fixed, we are able to derive the capital stock of the incumbent at time T explicitly and obtain [5]

$$\begin{aligned} K_1(T) = \Bigg(&\hat{K}_1^M + \frac{1}{N}\Bigg[-\hat{K}_1^M(\xi_1^M - \xi_2^M)e^{\xi_1^M T} \\ &-\frac{1}{2}(\hat{\mu}_1^M - \beta)(\xi_1^M - r - \delta)(\xi_2^M - r - \delta)(1 - e^{(\xi_1^M - \xi_2)T})\Bigg]\Bigg) \\ &\times\left(1 - \frac{1}{2N}(\xi_1^M - r - \delta)(\xi_2^M - r - \delta)(1 - e^{(\xi_1^M - \xi_2^M)T})\pi\right)^{-1}, \end{aligned} \tag{8}$$

where

$$N = (\xi_1^M - r - \delta) - (\xi_2^M - r - \delta)e^{(\xi_1^M - \xi_2^M)T}. \tag{9}$$

Here ξ_1^M and ξ_2^M denote the positive and negative eigenvalues of the steady state of the incumbent problem before entry takes place.

2.2. Analysis and economic interpretation (open-loop case)

After the entry time, the resulting differential game between two firms with open-loop information structure has a unique equilibrium, which turns out to be a saddle path (see Reynolds, 1987 for details). The resulting steady state capital stock is smaller than that of the monopolist and the one corresponding to the Markov perfect duopoly (see Reynolds, 1987 or Dragone et al., 2010), i.e.

$$\begin{aligned} \hat{K}_1^M &= \frac{A - b(r+\delta)}{2 + \delta c(r+\delta)} > \frac{A - b(r+\delta)}{3 + \delta c(r+\delta) - \frac{\sigma}{\pi - c(r+\delta)}} = \hat{K}_1^{\mathrm{MP}} \\ &> \frac{A - b(r+\delta)}{3 + \delta c(r+\delta)} = \hat{K}_1^O. \end{aligned} \tag{10}$$

At the end of the period prior to the entry of firm 2, firm 1 has the following concave salvage function, which equals the value function of the duopolistic period at that time (for a derivation we refer to Appendix B):

$$S(K_1) = Y_1 + Y_2 K_1 + Y_3 K_1^2$$

where $Y_1 > 0$, $Y_2 > 0$ and $Y_3 < 0$ (for the full expressions of these parameters we again refer to Appendix B).

[5] Take the explicit expression of the capital stock with finite time horizon ((40) of Appendix A) together with the salvage value implied by the Markov perfect commitment scenario (see Reynolds, 1987) and solve the equation with respect to $K_1(T)$. Note that in the open-loop scenario the investments of firm 2 (the competitor) are treated as functions of time only (i.e. firm 1 does not include the effect of the capital stocks on the optimal investments in the calculation of the optimal strategy).

By following the same steps (with different salvage value function) as in the Markov perfect equilibrium case, we again can calculate the capital stock of firm 1 at the entry time, i.e.

$$\begin{aligned} K_1(T) = \Bigg(&\hat{K}_1^M + \frac{1}{N}\Bigg[-\hat{K}_1^M(\xi_1^M - \xi_2^M)e^{\xi_1^M T} \\ &-\frac{1}{2}(\hat{\mu}_1^M - Y_2)(\xi_1^M - r - \delta)(\xi_2^M - r - \delta)(1 - e^{(\xi_1^M - \xi_2^M)T})\Bigg]\Bigg) \\ &\times\left(1 - \frac{1}{2N}(\xi_1^M - r - \delta)(\xi_2^M - r - \delta)(1 - e^{(\xi_1^M - \xi_2^M)T})2Y_3\right)^{-1}, \end{aligned} \tag{11}$$

where N is given by (9).

2.3. Comparison

This section compares the results of the Markov perfect and the open-loop solutions in the case of fixed entry time T. We choose the same parameter values as in Reynolds (1987), i.e.

$$r = 0.1, \quad \delta = 0.05, \quad c = 10, \quad b = 100, \quad A = 60. \tag{12}$$

These imply the following value function parameter values for the Markov perfect equilibrium:

$$\begin{aligned} &\alpha = 1769.8, \quad \beta = 179.0, \quad \gamma = -17.09, \\ &\pi = -3.26, \quad \sigma = -1.2, \quad \epsilon = 0.17, \end{aligned} \tag{13}$$

and for the open-loop case

$$A_1 = 0.3787, \quad B_1 = 48.4420, \quad C_1 = 1936 \tag{14}$$

where

$$A_2 K_1^2 + B_2 K_1 + C_2, \tag{15}$$

denotes firm 1's value function in the second period in the open-loop case (for the detailed expressions we refer to Appendix C).

Figs. 1 and 2 depict the time paths for both (above: Markov perfect, below: open-loop) scenarios. The left panels illustrate the capital stocks and the right ones the investments of both firms. Firm 2 enters either at $T = 25$ (Fig. 1) or at $T = 5$ (Fig. 2). Further, we compare the optimal investment behavior of the incumbent (black line) with how the incumbent should invest if it was unaware of a firm entering at some future time T[6]. The resulting "non-anticipative" investment behavior is denoted by the dashed line.

In case of a relatively late entry time T (Fig. 1), the incumbent approaches the monopolistic steady state (grey line) before it starts to anticipate firm 2's entry. However, anticipation is considerably different depending on the assumed commitment scenario. In the open-loop one the anticipation effect is positive, i.e. the incumbent overinvests before the entry time. In the Markov perfect equilibrium it is slightly negative, i.e. the incumbent follows very close the non-anticipative investment path, where it slightly underinvests before entry takes place.

In case of an earlier entry time (Fig. 2), the incumbent stays far below the monopolistic steady state, because there is not enough time to reach it before entry takes place. In the open-loop case the positive anticipation effect is that strong, that the capital stock increases considerably faster compared to the non-anticipative case and to the Markov perfect equilibrium.

The form of the anticipation (positive or negative) cannot be shown analytically. Instead, we numerically tested the ranges

[6] That is the incumbent behavior when $T = +\infty$ in (4).

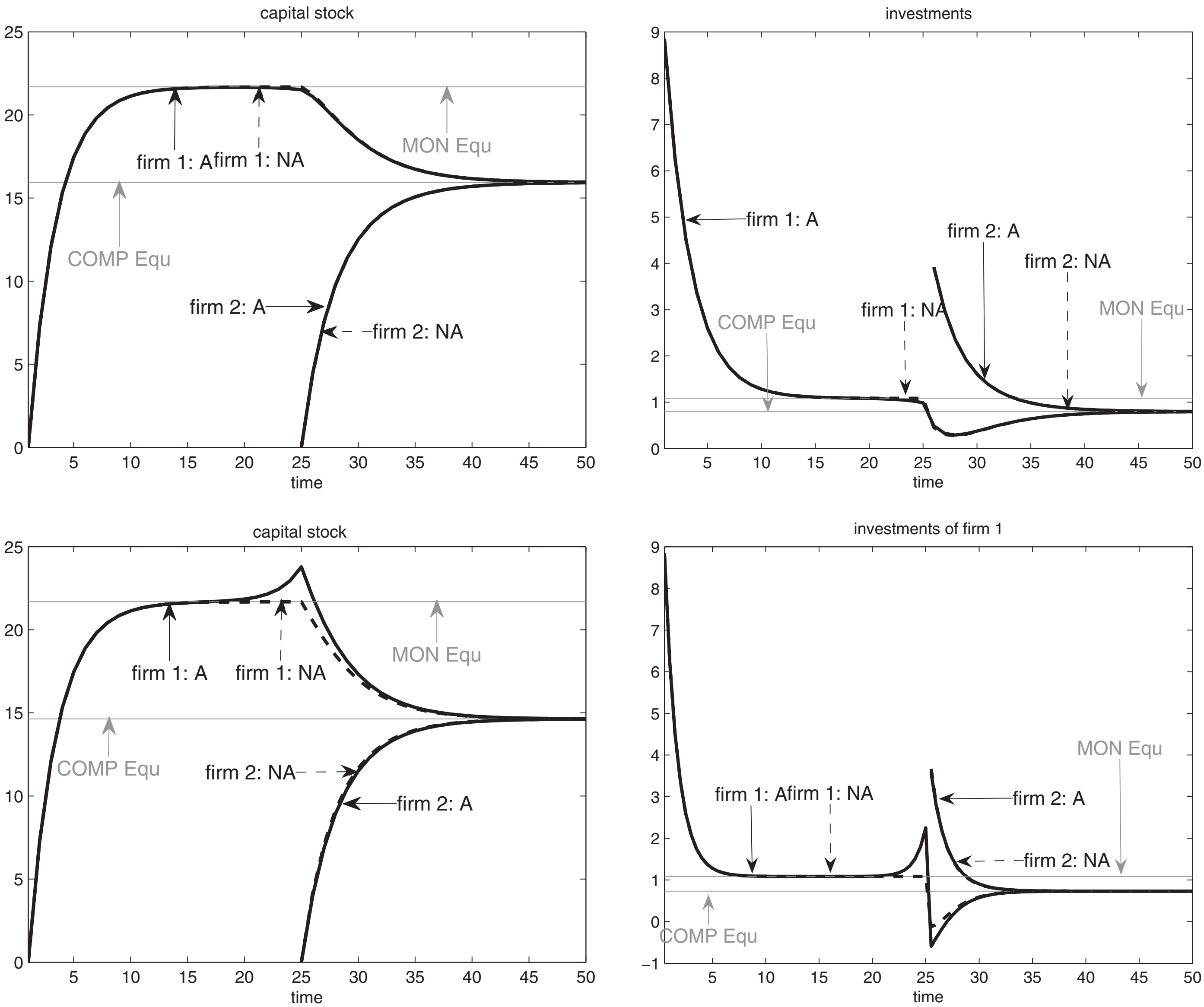

Fig. 1. Capital stocks and optimal investments of both firms over time (exogenous $T = 25$), above: Markov perfect, below: open-loop.

$r \in [0, 0.5]$, $\delta \in [0, 0.5]$, $c \in [0, 10]$, $b \in [0, 250]$ and $A \in [0, 60]$, and found that the just described behavior is robust against parameter changes and differs only in the magnitude.

The interpretation of the observed behavior is as follows. Under open-loop, firms cannot influence each other's capital stock development so directly as in the Markov perfect equilibrium. Therefore, a large K_1 at the entry time pays off for firm 1. Firm 2 has no direct means to negatively influence firm 1's investment. A large K_1 reduces firm 2's co-state so that firm 2's investments are also reduced. This will lead to a slower growth of firm 2 toward its steady state.

In the Markov perfect equilibrium there is a direct effect of capital stock of both firms on each other's investment. That implies that effects of an initial capital stock are less persistent. Firm 2 easily reduces firm 1's investment, because, according to Eq. (7), firm 2's capital stock directly enters the equation for firm 1's investment rate. Also profits in the Markov perfect equilibrium are lower due to increased strategic interactions formed by the over-investments of both firms. This all makes that there is less incentive to have a positive anticipation phase before the start of the duopoly period. The relatively lower profits corresponding to the Markov perfect equilibrium even result in a slightly negative anticipation phase.

3. The model with entry cost

In the sections before we assumed that the entry time T is fixed and entry is taken for granted (Section 2). Now we introduce positive entry costs F, and address the generalization such that from a certain time $\tilde{T}$ on, firm 2 can decide itself whether and when it wants to enter. If firm 2's payoff is not high enough to compensate for the entry costs, it will not enter the market. Firm 1 knows this and could adjust its strategy, i.e. overinvest, to deter entry. Therefore, firm 1's possibilities are (i) to accommodate entry like in the previous section, or (ii) to deter entry. It will choose that option that maximizes its own payoff. In the following two sections we present the Markov perfect equilibrium and the open-loop case, respectively. We end with a comparison of the two cases.

3.1. Analysis and interpretation (Markov perfect equilibrium)

In the Markov perfect equilibrium firm 2 has the following entry condition:

$$\begin{aligned} &V_2^{\mathrm{MP}} - F > 0 : \text{enter}, \\ &V_2^{\mathrm{MP}} - F \le 0 : \text{do not enter}, \end{aligned} \tag{16}$$

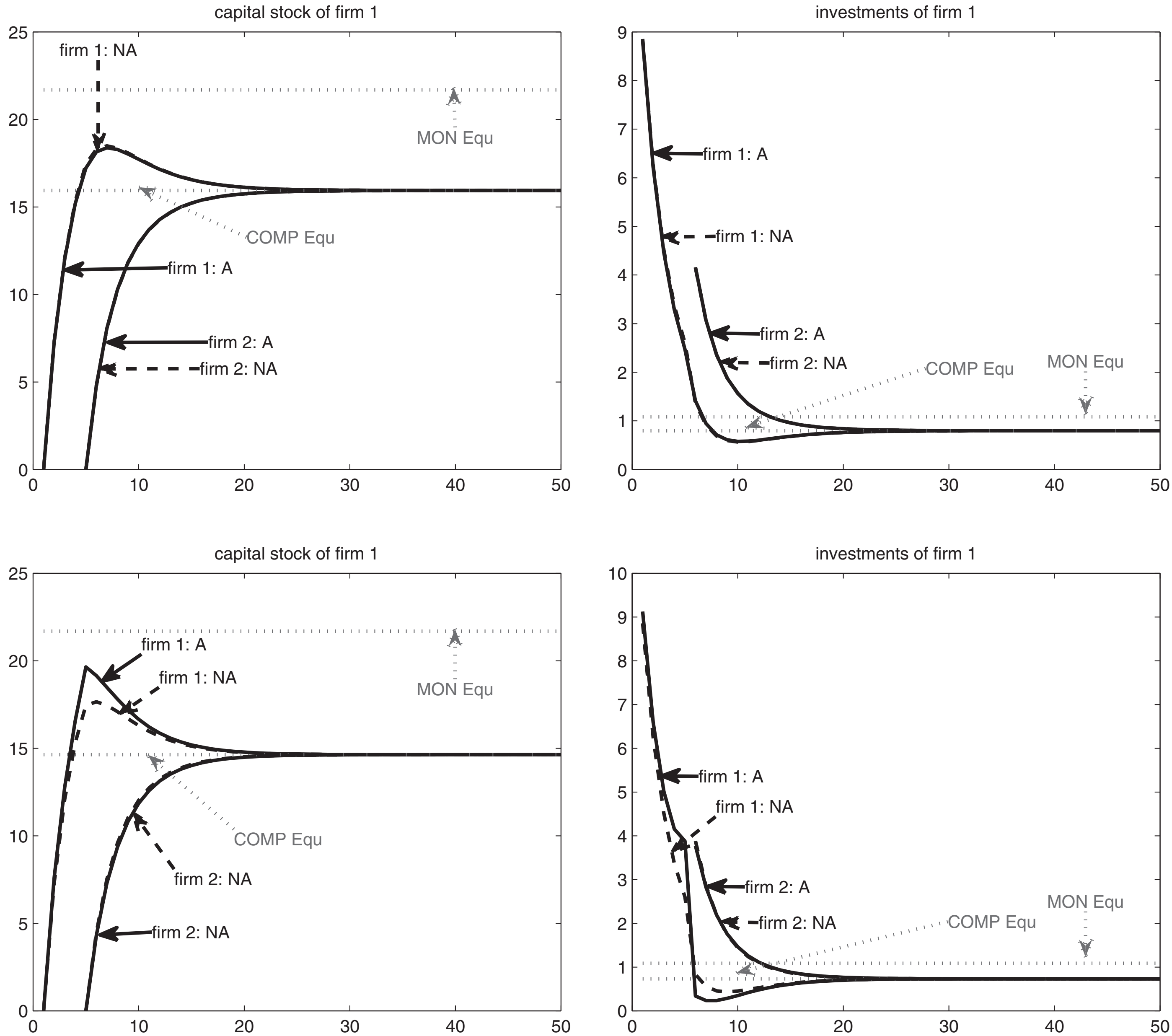

Fig. 2. Capital stocks and optimal investments of both firms over time (exogenous $T = 5$), above: Markov perfect, below: open-loop.

where V_2^{MP} denotes firm 2's value function at time t. The left hand side of (16) represents the value of firm 2 if it enters the market. If the value is positive (non-positive), it should enter (not enter).

Given firm 2's value function (see Reynolds, 1987), the above entry condition (16) at the entry time $\tilde{T}$ thus reads

$$\frac{\epsilon}{2}K_1^2 + \gamma K_1 + \alpha - F > 0, \tag{17}$$

in which firm 2's own capital stock does not occur, since $K_2(\tilde{T}) = 0$. For the parameter values (12), and also for the ranges $r \in [0, 0.5]$, $\delta \in [0, 0.5]$, $c \in [0, 10]$, $b \in [0, 250]$ and $A \in [0, 60]$, we obtain $\varepsilon > 0$, $\gamma < 0$ and $\alpha > 0$. The negative root,

$$K_1^{critMP} := \frac{1}{\epsilon}\left(-\gamma - \sqrt{\gamma^2 - 2\epsilon(\alpha - F)}\right) \tag{18}$$

denotes firm 1's capital stock level where firm 2's entry condition is binding. If firm 1 chooses a capital stock that is below K_1^{critMP}, firm 2 has a positive value of the second period and will enter the market. For $K_1 \geq K_1^{critMP}$ firm 2 will not enter.

Based on (18), two boundaries on the entry costs can be derived, i.e.

$$\underline{F}^{MP} := \alpha - \frac{\gamma^2}{2\epsilon}, \quad \overline{F}^{MP} := \alpha. \tag{19}$$

These boundaries are illustrated in Fig. 3, and can be interpreted as follows. The decreasing curve in the figure represents the value function of firm 2 depending on firm 1's capital stock at the time of possible entry, i.e. $\frac{\epsilon}{2}K_1^2 + \gamma K_1 + \alpha$. From the figure we can conclude that, if the entry costs are always smaller than $\underline{F}^{MP}$, firm 1 cannot do anything against firm 2's entry. If the entry costs are higher than $\underline{F}^{MP}$ firm 1 can turn firm 2's total payoff (value function minus the entry costs) negative, implying that firm 2 will not enter. However, if the entry costs are higher than firm 2's value function in case of a zero capital stock of firm 1 at the entry time, firm 2's total payoff will always be negative and will never enter the market. This boundary is plotted as $\overline{F}^{MP}$ in the above expression and in the figure. Hence, firm 1 has to choose between entry deterrence and accommodation

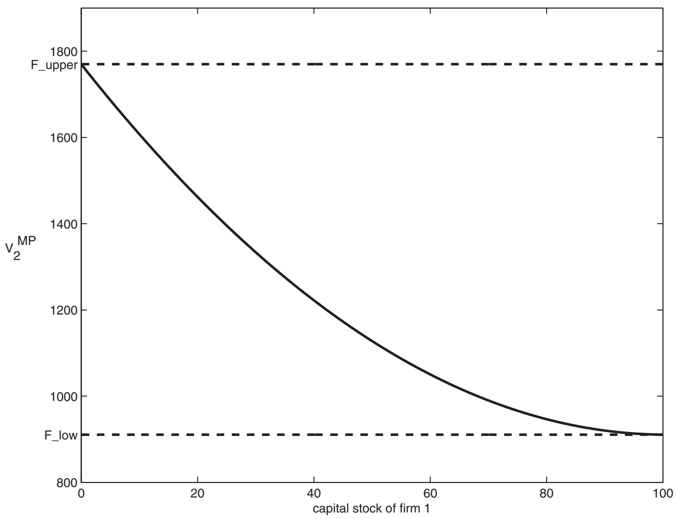

Fig. 3. Value function of firm 2 depending on firm 1's capital stock (Markov perfect case).

only if $\underline{F}^{\mathrm{MP}} \le F \le \overline{F}^{\mathrm{MP}}$. Here it has to be remarked that some entry cost level $F^M \in (\underline{F}^{\mathrm{MP}}, \overline{F}^{\mathrm{MP}})$ exists , such that for $F \in [F^M, \overline{F}^{\mathrm{MP}}]$ firm 2 already chooses not to enter if firm 1 just chooses the monopoly investment time path. For costs of entry belonging to this interval we have a scenario where firm 1 is a natural monopoly. However, for $F \in [\underline{F}^{\mathrm{MP}}, F^M)$ entry deterrence requires higher investments than what firm 1 would do in case of a monopoly, and the question is then whether this overinvestment is not too costly. For $F < \underline{F}^{\mathrm{MP}}$ firm 2 will always enter. On the other hand, for $F > \overline{F}^{\mathrm{MP}}$ firm 2 will never enter.

Let us define $K_1^M(\tilde{T})$ as the capital stock at $\tilde{T}$ if firm 1 would behave as a monopolist forever. In the current model the market structure develops over time in the following way:

- $K_1^M(\tilde{T}) \ge K^{\mathrm{critMP}}$: It is not necessary for the monopolist to adapt the monopolistic strategy, since firm 2 will not enter. This is the natural monopoly scenario that occurs when $F \ge F^M$.
- $K_1^M(\tilde{T}) < K^{\mathrm{critMP}}$: If firm 1 behaves as monopolist, firm 2 will enter (if $F < F^M$). Then firm 1 has to compare the payoffs resulting from (i) the case where it accommodates entry (like in the previous section) and firm 2 enters at $\tilde{T}$ with that from (ii) the firm deters entry, i.e. it increases the capital stock such that it equals K^{critMP} at time $\tilde{T}$, and does not fall below this level for $t > \tilde{T}$.

In order to find out whether it is profitable to raise its capital stock to K^{critMP} (such that firm 2 does not enter), it is necessary to calculate the corresponding value functions. Let $V_1^{\mathrm{MP},i}$ denote firm 1's value function when it does not raise its capital stock to K^{critMP}, or higher (firm 2 does enter the market). On the other hand, let $V_1^{\mathrm{MP},ii}$ denote firm 1's value function when it does raise its capital stock to that level (firm 2 does not enter). We derived the following analytical expressions:

$$
\begin{aligned}
V_1^{\mathrm{MP},i} &= -(A-\hat{K}_1^M)\hat{K}_1^M \frac{e^{-r\tilde{T}}-1}{r} - (A-2\hat{K}_1^M)\hat{K}_1^M \frac{e^{(\xi_2^M-r)\tilde{T}}-1}{\xi_2^M-r} \\
&\quad - (\hat{K}_1^M)^2 \frac{e^{(2\xi_2^M-r)\tilde{T}}-1}{2\xi_2^M-r} \\
&\quad - b\left(-\delta\hat{K}_1^M \frac{e^{-r\tilde{T}}-1}{r} - \frac{2\hat{K}_1^M}{c(\xi_2^M-r-\delta)} \frac{e^{(\xi_2^M-r)\tilde{T}}-1}{\xi_2^M-r}\right) \\
&\quad - \frac{c}{2}\left(-\delta^2(\hat{K}_1^M)^2 \frac{e^{-r\tilde{T}}-1}{r} - 4\delta \frac{(\hat{K}_1^M)^2}{c(\xi_2^M-r-\delta)} \frac{e^{(\xi_2^M-r)\tilde{T}}-1}{\xi_2^M-r}\right. \\
&\quad \left. + \frac{2(\hat{K}_1^M)^2}{c^2(\xi_2^M-r-\delta)^2} \frac{e^{(2\xi_2^M-r)\tilde{T}}-1}{2\xi_2^M-r}\right) \\
&\quad + e^{-r\tilde{T}}\left(\frac{\pi}{2}\left(\hat{K}_1^M(1-e^{\xi_2^M\tilde{T}})\right)^2 + \beta\left(\hat{K}_1^M(1-e^{\xi_2^M\tilde{T}})\right) + \alpha\right) \\
V_1^{\mathrm{MP},ii} &= -(A-\hat{K}_1^M)\hat{K}_1^M \frac{e^{-r\tilde{T}}-1}{r} + (A-2\hat{K}_1^M)\frac{\bar{c}_1(\xi_1^M-r-\delta)}{2} \\
&\quad \times \frac{e^{(\xi_1^M-r)\tilde{T}}-1}{\xi_1^M-r} + (A-2\hat{K}_1^M)\frac{\bar{c}_2(\xi_2^M-r-\delta)}{2}\frac{e^{(\xi_2^M-r)\tilde{T}}-1}{\xi_2^M-r} \\
&\quad - \frac{\bar{c}_1^2(\xi_1^M-r-\delta)^2}{4}\frac{e^{(2\xi_1^M-r)\tilde{T}}-1}{2\xi_1^M-r} - \frac{\bar{c}_2^2(\xi_2^M-r-\delta)^2}{4} \\
&\quad \times \frac{e^{(2\xi_2^M-r)\tilde{T}}-1}{2\xi_2^M-r} - \frac{\bar{c}_1\bar{c}_2(\xi_1^M-r-\delta)(\xi_2^M-r-\delta)}{2} \\
&\quad \times \frac{e^{(\xi_1^M+\xi_2^M-r)\tilde{T}}-1}{\xi_1^M+\xi_2^M-r} - b\left(-c\delta\hat{K}_1^M\frac{e^{-r\tilde{T}}-1}{r} + \bar{c}_1\frac{e^{(\xi_1^M-r)\tilde{T}}-1}{\xi_1^M-r}\right. \\
&\quad \left. + \bar{c}_2\frac{e^{(\xi_2^M-r)\tilde{T}}-1}{\xi_2^M-r}\right) - \frac{1}{2c}\left(-c^2\delta^2(\hat{K}_1^M)^2\frac{e^{-r\tilde{T}}-1}{r}\right. \\
&\quad + 2c\delta\hat{K}_1^M\bar{c}_1\frac{e^{(\xi_1^M-r)\tilde{T}}-1}{\xi_1^M-r} + 2c\delta\hat{K}_1^M\bar{c}_2\frac{e^{(\xi_2^M-r)\tilde{T}}-1}{\xi_2^M-r} \\
&\quad \left. + \bar{c}_1^2\frac{e^{(2\xi_1^M-r)\tilde{T}}-1}{2\xi_1^M-r} + \bar{c}_2^2\frac{e^{(2\xi_2^M-r)\tilde{T}}-1}{2\xi_2^M-r} + 2\bar{c}_1\bar{c}_2\frac{e^{(\xi_1^M+\xi_2^M-r)\tilde{T}}-1}{\xi_1^M+\xi_2^M-r}\right) \\
&\quad + \frac{e^{-r\tilde{T}}}{r}\left((A-K^{\mathrm{critMP}})K^{\mathrm{critMP}} - b\delta K^{\mathrm{critMP}} - \frac{c\delta^2}{2}(K^{\mathrm{critMP}})^2\right)
\end{aligned}
\tag{20}
$$

where $\bar{c}_i$ (for $i = 1, 2$) are functions of the critical capital stock derived in (23), i.e. $\bar{c}_i = \bar{c}_i(K^{\mathrm{critMP}})$, for $i = 1, 2$, (see Appendix C).

Let us consider the market situation for the same parameter values as in the benchmark scenario (see p. 8). Then we get $\underline{F}^{\mathrm{MP}} = 910$ and $\overline{F}^{\mathrm{MP}} = 1769.8$. If the entry costs are between these two boundaries, firm 1 must choose whether to deter or to accommodate entry. Table 1 summarizes the optimal actions of both firms and the market implications for different values of the entry cost. If $F > \underline{F}^{\mathrm{MP}}$, firm 1 is in principle able to prevent firm 2 from a market entry. However, if F is slightly above $\underline{F}^{\mathrm{MP}}$, that is too costly, because it requires too much investments. At $F_I^{\mathrm{MP}} = 1333.2$, firm 1 is indifferent between entry accommodation and entry deterrence, i.e. raising the capital stock to K^{critMP}. For $F > F_I^{\mathrm{MP}}$ it is profitable for firm 1 to deter entry. Hence, firm 1 is a monopolist, but a constrained monopolist in the sense that capital stock must be large enough to prevent entry of the other firm. The smaller is F, the more profitable is entry for firm 2, which implies that the more constrained the incumbent is, i.e. the larger is the lower bound of the capital stock that deters entry.

The long run optimal steady state capital stock for the situation that arises after $\tilde{T}$ is depicted in Fig. 4 for different entry costs F, where $\tilde{T} = 25$. Analogous to the table we see that for low F firm 2 enters and both firms will admit the steady state capital stock of the duopoly under the Markov perfect scenario. For $F = F_I^{\mathrm{MP}}$ firm 1 is indifferent between entry deterrence and entry accommodation. This implies that both steady state capital stocks are optimal, i.e. that of the duopoly and K^{critMP} (the minimal level needed to deter entry). For $F > F_I^{\mathrm{MP}}$ first the steady state capital stock equals K^{critMP}, which is a decreasing function of F. For even higher values of F, the curve of the critical capital stock crosses the steady state of the monopolistic scenario (i.e. $K^{\mathrm{critMP}} = K^M$). For values of F higher than where $K^{\mathrm{critMP}} = K^M$, firm 1 is a natural monopoly. This means that it is optimal to behave as a monopolist, since this is enough to deter entry. To summarize, we observe that firm 1 accommodates entry for low values of F, deters entry for intermediate values of F, and is a natural monopolist for high values of F.

Table 1
Optimal actions of both firms for different entry costs ($\tilde{T} = 25$, $K_{10} = 0$, Markov perfect).

F	K^{critMP}	Implication
<910	–	Firm 2 will enter
1200	46.1973	Increasing the $K_1(\tilde{T})$ to K^{critMP} is too expensive, firm 2 will enter
1300	32.8603	Increasing the $K_1(\tilde{T})$ to K^{critMP} is too expensive, firm 2 will enter
1333.2	30.0334	Firm 1 is indifferent
1350	28.6452	Firm 1 increases the $K_1(\tilde{T})$ to K^{critMP} , firm 2 will not enter
1400	24.6639	Firm 1 increases the $K_1(\tilde{T})$ to K^{critMP} , firm 2 will not enter
1450	20.8814	Firm 1 behaves as *usual* monopolist, firm 2 will never enter
1550	13.8099	Firm 1 behaves as *usual* monopolist, firm 2 will never enter
>1769.8	–	Firm 1 behaves as *usual* monopolist, firm 2 will never enter

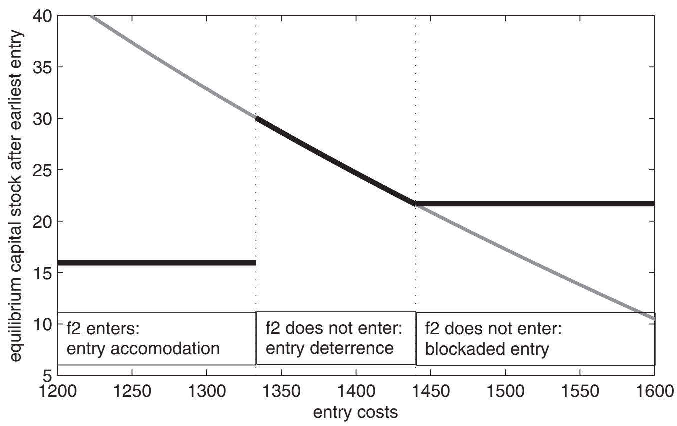

Fig. 4. Equilibrium capital stocks after the switching time $\tilde{T}$ for different entry costs F (Markov perfect).

3.2. Analysis and interpretation (open-loop case)

For the open-loop case firm 2 has the following entry condition:

$$\begin{aligned} &V_2^O - F > 0 : \text{enter}, \\ &V_2^O - F \leq 0 : \text{do not enter}, \end{aligned} \tag{21}$$

where V_2^O denotes firm 2's open-loop value function at t. The interpretation is analogous to that of the Markov perfect equilibrium case. As firm 2 starts with a zero capital stock, the left hand side only depends on the capital stock of firm 1. It is straightforward[7] to transform the lhs of (21) into a quadratic form in $K_1(\tilde{T})$, i.e.

$$A_2K_1^2 + B_2K_1 + C_2 - F. \tag{22}$$

The negative root of (22),

$$K^{\text{critOL}} := \frac{1}{2A_2}\Big(-B_2 - \sqrt{B_2^2 - 4A_2(C_2 - F)}\Big) \tag{23}$$

denotes the critical capital stock value of firm 1 where firm 2's entry condition is binding. For the parameter values (12), and also for the ranges $r \in [0, 0.5]$, $\delta \in [0, 0.5]$, $c \in [0, 10]$, $b \in [0, 250]$ and $A \in [0, 60]$, we obtain $A_2 > 0$, $B_2 < 0$ and $C_2 > 0$, implying that the lhs of (22) is a U-shaped function. By using the above form (23) we again obtain two boundaries on the entry costs, i.e.

$$\underline{F}^{\text{OL}} := C_2 - \frac{B_2^2}{4A_2}, \quad \overline{F}^{\text{OL}} := C_2. \tag{24}$$

The interpretation is analogous to the Markov perfect equilibrium, only the levels are different.

The value functions of firm 1 for the case where the capital stock is increased to K^{critOL} (in order to deter entry) and for the case where firm 1 accommodates entry (i.e. firm 2 enters the market) read ($V_1^{O,i}$ when it accommodates entry, $V_1^{O,ii}$ when it deters entry):

$$\begin{aligned} V_1^{O,i} = &-(A - \hat{K}_1^M)\hat{K}_1^M \frac{e^{-r\tilde{T}} - 1}{r} - (A - 2\hat{K}_1^M)\hat{K}_1^M \frac{e^{(\xi_2^M - r)\tilde{T}} - 1}{\xi_2^M - r} \\ &- \left(\hat{K}_1^M\right)^2 \frac{e^{(2\xi_2^M - r)\tilde{T}} - 1}{2\xi_2^M - r} \\ &- b\left(-\delta\hat{K}_1^M \frac{e^{-r\tilde{T}} - 1}{r} - \frac{2\hat{K}_1^M}{c(\xi_2^M - r - \delta)} \frac{e^{(\xi_2^M - r)\tilde{T}} - 1}{\xi_2^M - r}\right) \\ &- \frac{c}{2}\left(-\delta^2\left(\hat{K}_1^M\right)^2 \frac{e^{-r\tilde{T}} - 1}{r} - 4\delta \frac{\left(\hat{K}_1^M\right)^2}{c(\xi_2^M - r - \delta)} \frac{e^{(\xi_2^M - r)\tilde{T}} - 1}{\xi_2^M - r}\right. \\ &\left. + \frac{2\left(\hat{K}_1^M\right)^2}{c^2\left(\xi_2^M - r - \delta\right)^2} \frac{e^{(2\xi_2^M - r)\tilde{T}} - 1}{2\xi_2^M - r}\right) + e^{-r\tilde{T}}\Big(A_1\big(\hat{K}_1^M(1 - e^{\xi_2^M\tilde{T}})\big)^2 \\ &+ B_1\big(\hat{K}_1^M(1 - e^{\xi_2^M\tilde{T}})\big) + C_1\Big) \\ V_1^{O,ii} = &-(A - \hat{K}_1^M)\hat{K}_1^M \frac{e^{-r\tilde{T}} - 1}{r} + (A - 2\hat{K}_1^M)\frac{\bar{c}_1(\xi_1^M - r - \delta)}{2} \\ &\times \frac{e^{(\xi_1^M - r)\tilde{T}} - 1}{\xi_1^M - r} + (A - 2\hat{K}_1^M)\frac{\bar{c}_2(\xi_2^M - r - \delta)}{2}\frac{e^{(\xi_2^M - r)\tilde{T}} - 1}{\xi_2^M - r} \\ &- \frac{\bar{c}_1^2(\xi_1^M - r - \delta)^2}{4}\frac{e^{(2\xi_1^M - r)\tilde{T}} - 1}{2\xi_1^M - r} - \frac{\bar{c}_2^2(\xi_2^M - r - \delta)^2}{4}\frac{e^{(2\xi_2^M - r)\tilde{T}} - 1}{2\xi_2^M - r} \\ &- \frac{\bar{c}_1\bar{c}_2(\xi_1^M - r - \delta)(\xi_2^M - r - \delta)}{2}\frac{e^{(\xi_1^M + \xi_2^M - r)\tilde{T}} - 1}{\xi_1^M + \xi_2^M - r} \\ &- b\left(-c\delta\hat{K}_1^M\frac{e^{-r\tilde{T}} - 1}{r} + \bar{c}_1\frac{e^{(\xi_1^M - r)\tilde{T}} - 1}{\xi_1^M - r} + \bar{c}_2\frac{e^{(\xi_2^M - r)\tilde{T}} - 1}{\xi_2^M - r}\right) \\ &- \frac{1}{2c}\left(-c^2\delta^2\left(\hat{K}_1^M\right)^2\frac{e^{-r\tilde{T}} - 1}{r} + 2c\delta\hat{K}_1^M\bar{c}_1\frac{e^{(\xi_1^M - r)\tilde{T}} - 1}{\xi_1^M - r}\right. \\ &+ 2c\delta\hat{K}_1^M\bar{c}_2\frac{e^{(\xi_2^M - r)\tilde{T}} - 1}{\xi_2^M - r} + \bar{c}_1^2\frac{e^{(2\xi_1^M - r)\tilde{T}} - 1}{2\xi_1^M - r} + \bar{c}_2^2\frac{e^{(2\xi_2^M - r)\tilde{T}} - 1}{2\xi_2^M - r} \\ &\left. + 2\bar{c}_1\bar{c}_2\frac{e^{(\xi_1^M + \xi_2^M - r)\tilde{T}} - 1}{\xi_1^M + \xi_2^M - r}\right) \\ &\frac{e^{-r\tilde{T}}}{r}\left((A - K^{\text{critOL}})K^{\text{critOL}} - b\delta K^{\text{critOL}} - \frac{c\delta^2}{2}(K^{\text{critOL}})^2\right) \end{aligned} \tag{25}$$

where $\bar{c}_i$ (for $i = 1, 2$) are functions of the critical capital stock derived in (23), i.e. $\bar{c}_i = \bar{c}_i(K^{\text{critOL}})$ (for $i = 1, 2$). As before, the complete expressions for the derived value function parameters can be found in Appendix C.

We again adopt the parameter values presented in (12). The boundaries for the entry cost equal

$$\underline{F}^{\text{OL}} = 1490.5 \quad \text{and } \overline{F}^{\text{OL}} = 1936. \tag{26}$$

The optimal actions by both firms for several choices of F are summarized in Table 2. Here possible entry can take place from $\tilde{T} = 25$

[7] The way to derive the value function is straightforward, but requires very tedious analysis. The exact values of the parameters can be found in Appendix C.

Table 2
Optimal actions of both firms for different entry costs ($\tilde{T} = 25$, $K_{10} = 0$, open-loop).

F	$K^{\text{crit OL}}$	Implication
<1490.5	–	Firm 2 will enter
1500	46.3243	Increasing the $K_1(\tilde{T})$ to $K^{\text{crit OL}}$ is too expensive, firm 2 will enter
1524.1	39, 3518	Firm 1 is indifferent
1550	34.4254	Firm 1 increases the $K_1(\tilde{T})$ to $K^{\text{crit OL}}$, firm 2 will not enter
1600	27.3562	Firm 1 increases the $K_1(\tilde{T})$ to $K^{\text{crit OL}}$, firm 2 will not enter
1650	21.7915	Firm 1 increases the $K_1(\tilde{T})$ to $K^{\text{crit OL}}$, firm 2 will not enter
1700	17.0499	Firm 1 behaves as *usual* monopolist, firm 2 will never enter
1800	9.0344	Firm 1 behaves as *usual* monopolist, firm 2 will never enter
1900	2.2397	Firm 1 behaves as *usual* monopolist, firm 2 will never enter
>1936	–	Firm 1 behaves as *usual* monopolist, firm 2 will never enter

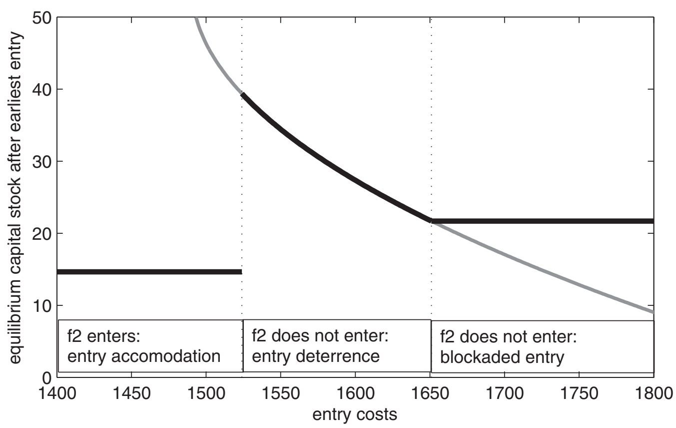

Fig. 5. Equilibrium capital stocks after the switching time $\tilde{T}$ for different entry costs F (open-loop).

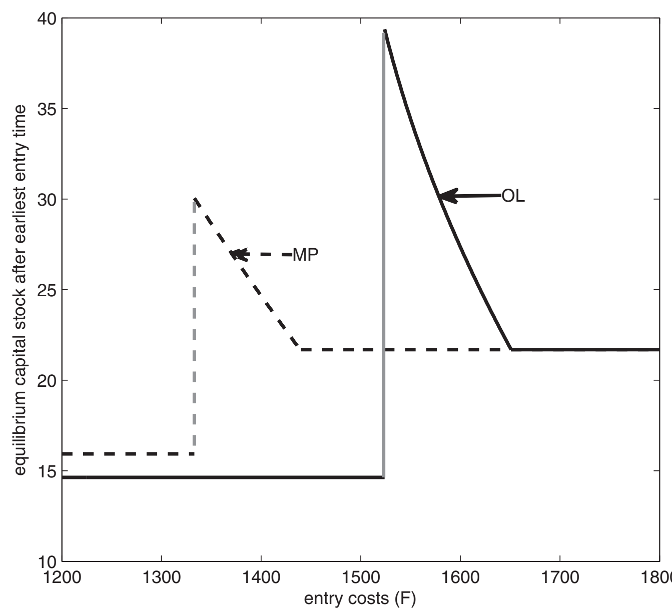

Fig. 6. Equilibrium capital stocks after the switching time $\tilde{T}$ for different entry costs F.

onwards. The behavior is analogous to the Markov perfect equilibrium case. The only difference is that both $\underline{F}^{\text{OL}}$ and $\overline{F}^{\text{OL}}$ are higher than in the Markov perfect equilibrium. The reason for this is that, if the duopoly has an open-loop structure, profits for the firms are higher. Consequently, the entry condition switches for higher entry costs F, compared to the Markov perfect equilibrium. Or, to put it differently, for given F firm 1 needs to overinvest more to deter entry of firm 2. Therefore, it starts doing this for larger values of F and needs to keep on doing it for larger values of F than in the Markov perfect equilibrium. Fig. 5, having the same qualitative characteristics as Fig. 4 for the Markov perfect equilibrium, depicts long run steady state values of firm 1 as a function of F for the different cases.

3.3. Comparison

The difference between the Markov perfect and the open-loop case is illustrated in Fig. 6, where the long-run steady state capital stock of firm 1 is depicted for different entry costs F (Markov perfect: dashed line, open-loop: solid line). If the entry costs are very low, firm 2 will definitely enter. In that case the steady state capital stock is higher in the Markov perfect equilibrium due to strategic interactions resulting in higher overinvestments (see Reynolds, 1987). For values of F (slightly) larger than F_I^{MP} it is optimal for the incumbent to increase its capital stock to K^{critMP}, and, as explained before, K^{critMP} is decreasing in F. When K^{critMP} gets smaller than the monopolistic steady state capital stock $\hat{K}_1^M$, it is optimal for firm 1 to behave as a usual unconstrained monopolist.

If we compare open-loop to Markov perfect, we know that in the latter case profits are lower due to strategic interactions resulting in higher overinvestments. This implies that in the case of open-loop an entry deterrence policy requires that the incumbent should acquire a larger capital stock (K^{critOL}) to keep firm 2 out of the market. This makes that entry deterrence is only profitable for larger values of F, and this results in the existence of an F-domain $(F_I^{\text{OL}}, F^{M,\text{OL}})$ where under open-loop firm 1 has a large capital stock to deter entry, while under Markov perfect firm 1 already is a natural monopoly. As a result, for $F \in (F_I^{\text{OL}}, F^{M,\text{OL}})$ the steady state capital stock under open-loop is higher than under Markov perfect. Usually in duopoly the Markov perfect capital stock exceeds the open-loop one. The threat of a new entry reverses this well known relationship here.

In the analysis before we have assumed that firm 2 can enter from $\tilde{T} = 25$ onwards. We now address the question what happens if $\tilde{T}$ changes. The result is plotted in Fig. 7. In this figure $K^{\text{crit}a}(\tilde{T})$ ($a \in \{\text{MP,OL}\}$) represents the maximal capital stock firm 1 wants to build up to prevent entry. Then $F_I^a(\tilde{T})$ is the entry cost where firm 2 is indifferent between entry and non-entry at time $\tilde{T}$, given that firm 1's capital stock equals $K^{\text{crit}a}(\tilde{T})$. So, first $K^{\text{crit}a}(\tilde{T})$ is determined, and based on that we establish $F_I^a(\tilde{T})$.

The dashed lines represent the critical capital stock $K^{\text{crit}a}$ as a function of $\tilde{T}$, whereas the solid lines denote the corresponding entry costs F_I^a. Obviously, the dependence on $\tilde{T}$ is non-trivial. $K^{\text{crit}a}$ is hump-shaped and the corresponding F_I^a U-shaped between $\tilde{T} = 0$ and approximately $\tilde{T} = 40$. The following two effects are responsible:

(i) Investment cost effect: First, consider the case $\tilde{T} = 0$. Since we have zero initial capital stock for the incumbent, i.e. $K_{10} = 0$, firm 2 will enter whenever $F < \overline{F}^a$. If $\tilde{T}$ goes up, firm 1 has the time to invest and build up a capital stock before eventual entry takes place. Note that a fast growth of K_1 is costly due

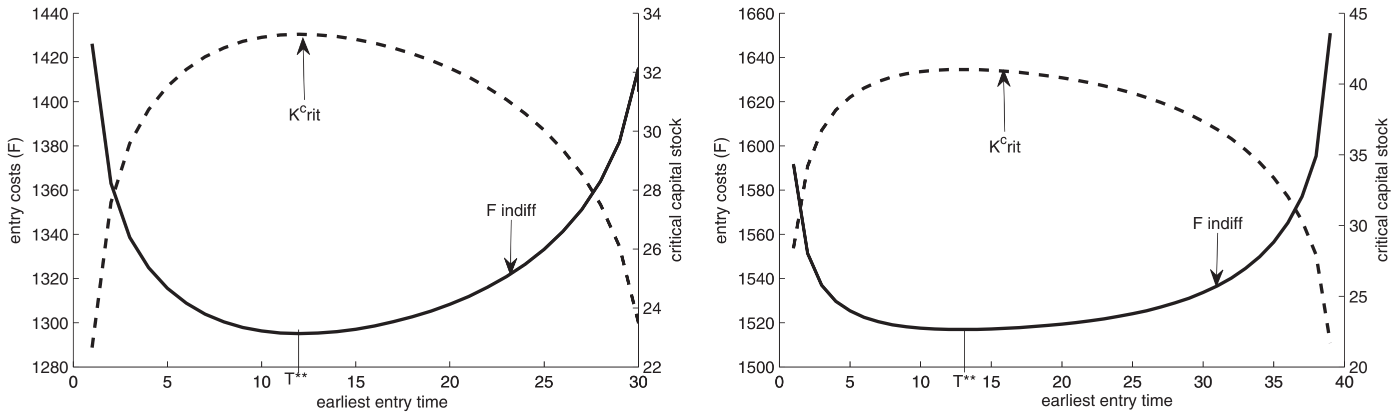

Fig. 7. Dependence of the switching costs (where firm 1 is indifferent) and the corresponding K^a ($a \in \{\mathrm{CFB}, \mathrm{COL}\}$) on $\tilde{T}$ (rhs: Markov perfect, lhs: open-loop).

to convex adjustment costs. The larger $\tilde{T}$, the cheaper for firm 1 to reach a certain capital stock value, which explains why $K^{\mathrm{crit}a}(\tilde{T})$ is increasing for low values of $\tilde{T}$. Since firm 2's payoff after entry is decreasing in both F and K_1, the F for which firm 2 is indifferent between entry and non-entry, is decreasing in $K^{\mathrm{crit}a}(\tilde{T})$. So $K^{\mathrm{crit}a}(\tilde{T})$ being increasing for small $\tilde{T}$ implies that $F_I^a(\tilde{T})$ is decreasing there.

(ii) Discounting effect: This effect results in a small $K^{\mathrm{crit}a}(\tilde{T})$ when $\tilde{T}$ is large. This is because entry far in the future does not affect firm 1's payoff that much due to discounting. This implies that the incentive for firm 1 to incur additional investment costs is low. Hence, $K^{\mathrm{crit}a}(\tilde{T})$ is decreasing in $\tilde{T}$ when $\tilde{T}$ is large. Since, as explained above, F_I^a is a decreasing function of $K^{\mathrm{crit}a}$, $F_I^a(\tilde{T})$ is increasing when $\tilde{T}$ is large.

Taking both effects together results in the U-shaped form of F_I^a. For early potential entry times the investment cost effect dominates, and after that the discount effect is stronger. If $\tilde{T}$ exceeds a certain time (approximately 30 in the Markov perfect equilibrium, and 40 in the open-loop case) the corresponding $K^{\mathrm{crit}a}$ falls below the long-run capital stock level of the monopolist. Then we are in the natural monopoly scenario, where it is not necessary for the incumbent to adapt its strategy in order to prevent entry.

Comparing the Markov perfect equilibrium with the open-loop one again shows that the maximal entry cost where firm 2 wants to enter, is higher for the open-loop case. The reason for that is as discussed before: open-loop gives higher duopoly profits.

Fig. 7 tells the incumbent under all circumstances what is optimal to do: entry deterrence or entry accommodation. If $\tilde{T}$ is such that $K^{\mathrm{crit}a}$ is increasing, the incumbent simply compares the entry cost to $F_I^a(\tilde{T})$. If $F > F_I^a(\tilde{T})$, the incumbent deters entry. To do so it establishes which F_I^a is equal to F, and then determines which $K^{\mathrm{crit}a}$ corresponds to this F_I^a. The incumbent can now deter entry by building up K such that $K(\tilde{T}) = K^{\mathrm{crit}a}(F_I^a)$. For $F < F_I^a(\tilde{T})$, the corresponding $K^{\mathrm{crit}a}(\tilde{T})$ is too high, implying that it is too costly to increase capital stock to that level. Consequently, the incumbent accommodates entry.

If $\tilde{T}$ is such that $K^{\mathrm{crit}a}$ is decreasing, i.e. $\tilde{T} > \tilde{T}_{\max}$, the incumbent first determines which F_I^a is equal to the current entry cost F. Then it establishes which $K^{\mathrm{crit}a}$ corresponds to this F_I^a, because in this way it knows which capital stock level needs to be build up such that firm 2 will not enter. If this $K^{\mathrm{crit}a}$ exceeds $K^{\mathrm{crit}a}(\tilde{T}_{\max})$, the incumbent accommodates entry. However, if $K^{\mathrm{crit}a}$ is smaller than $K^{\mathrm{crit}a}(\tilde{T}_{\max})$, the incumbent deters entry. Here one should realize that, as time passes, the time it lasts until firm 2 may enter, which we denote by $\tilde{T}(t)$, decreases. It holds that $\tilde{T}(t) = \tilde{T} - t$. This implies that, indeed, $\tilde{T}(t)$ decreases over time and becomes equal to $\tilde{T}_{\max}$ at time $t = \tilde{T} - \tilde{T}_{\max}$. And at this point in time the incumbent wants to invest such that maximally $K^{\mathrm{crit}a}(\tilde{T}_{\max})$ is the capital stock at $\tilde{T}$, if this deters entry.

4. Conclusions

The paper considers the problem of a firm that is currently a monopolist but where in the future another firm can enter the market and offer the same product. As soon as this happens, firms will compete with the result that profits of the incumbent firm will diminish. In particular, the topic of this paper is how the current monopolist will anticipate this future entry threat.

Essentially there are two strategies for the incumbent to deal with a future entry threat. First, the firm can deter entry. This requires huge investments to increase production capacity. Consequently the firm raises output, which reduces output price, which in turn makes the market less profitable for a potential entrant. This strategy is in particular successful when the entry threat occurs far in the future so that there is time enough to gradually build up the capital stock level necessary to deter entry, and/or when the entry cost for the entrant is relatively large.

When the entry threat will occur relatively soon, and/or the cost to enter is relatively small, entry deterrence requires too much investments to be profitable. This implies that the incumbent has to accommodate entry. It depends on the commitment structure of the underlying differential game how the incumbent will anticipate future entry. In case of an open-loop differential game the incumbent overinvests prior to the entry occurrence. This is because a higher incumbent capital stock raises the incumbent's sales. This reduces output price, implying that the entrant will invest less in this market.

When we consider a Markov perfect equilibrium, the incumbent slightly underinvests prior to entry. This is because in such an equilibrium a higher market share is less persistent, because investment rates are directly influenced by capital stocks of the own firm and of the competitor. The entrant just has to increase its own capital stock in order to reduce investments, and thus the increase of the capital stock, of the incumbent. A second reason for anticipatory underinvestment by the incumbent is that profits are lower in a Markov perfect equilibrium. This reduces the incentive to invest in this market.

Our paper analyzes a case where market size is constant. A growing trend would in fact work out on the firms' investment decisions as if the discount rate would be lower. This will result in higher firm values, implying that entry deterrence will be a policy more worthwhile to pursue. A shrinking market share will have the opposite effect, i.e. this will decrease the discount rate, so that the current instantaneous profit will have a larger effect on the firms payoff. This implies that the firm is less willing to make extra immediate costs associated with overinvestment. Hence, a decreasing market size will pursue a policy of entry accommodation.

One important extension is to incorporate uncertainty in the analysis, which is commonly present in the real world. From Huisman and Kort (in press) we know that more uncertainty enhances a policy of entry deterrence. However, in their framework the firms are only allowed to invest once in a lumpy fashion. The current papers framework allows firms to invest whenever they want, resulting in various incremental changes of the capital stock. It would be interesting to establish whether the Huisman–Kort result still holds when our setting also includes uncertainty.

Another interesting topic for future research is to include innovation, which is an important means of competition in today's economy. A recent contribution is Dawid, Kopel, and Kort (2013), who study a differential duopoly game with symmetric firms, where at some unknown point in the future a product innovation can be implemented. It would be interesting to incorporate product innovation in our incumbent-entrant framework. In Dawid et al. (2013) innovation arrival rates are exogenously given. It seems worthwhile to endogenize the innovation process by taking R&D investments explicitly into account.

Appendix A. Solution of the monopolistic period for exogenous *T*

Here we have a usual optimal control model that can be used by the standard method. The Hamiltonian of problem (4) reads (we ignore the time argument whenever it this does not lead to confusion)

$$\mathcal{H}^{\text{mon}} = (A - K_1)K_1 - bI_1 - \frac{c}{2}I_1^2 + \mu^1(I_1 - \delta K_1) \tag{27}$$

where μ^1 denotes the adjoint variable for K_1. The first order condition with respect to the control (investments) equals

$$\mathcal{H}^{\text{mon}}_{I_1} = -b - cI_1 + \mu_1 = 0 \tag{28}$$

implying the following investment behavior

$$I_1 = \frac{\mu_1 - b}{c}. \tag{29}$$

Finally the adjoint equation with respect to capital stock reads

$$\dot{\mu}_1 = (r + \delta)\mu_1 - A + 2K_1. \tag{30}$$

Using (29) in the capital dynamics, the optimal behavior is described by the dynamical system (1) and (30), for which we can derive the following unique equilibrium values

$$\hat{K}_1^M = \frac{A - (r+\delta)b}{2 + (r+\delta)\delta c} \tag{31}$$

$$\hat{\mu}_1^M = \frac{Ac\delta + 2b}{2 + (r+\delta)\delta c} \tag{32}$$

$$\hat{I}_1^M = \frac{(A - (r+\delta)b)\delta}{2 + (r+\delta)\delta c} \tag{33}$$

Linearizing the system around this equilibrium yields the following eigenvalues of the Jacobian

$$\xi^1 = \frac{r + \sqrt{r^2 + 4((r+\delta)\delta + \frac{2}{c})}}{2} \tag{34}$$

$$\xi^2 = \frac{r - \sqrt{r^2 + 4((r+\delta)\delta + \frac{2}{c})}}{2} \tag{35}$$

Since $4((r+\delta)\delta + \frac{2}{c}) > 0$ and $r > 0$ it is obvious that $\xi^1 > 0$ and $\xi^2 < 0$ (both real). Thus the equilibrium is a saddle point.

In the full model presented in Section 2 the salvage value is determined by the value function of the second period, i.e. $\mu^1(T) = S_{K_1}(\cdot)$. Since the system is of first order the stable path can be calculated analytically. The general solution of the model equals

$$\begin{pmatrix} K_1 \\ \mu_1 \end{pmatrix} = \begin{pmatrix} \hat{K}_1^M \\ \hat{\mu}_1^M \end{pmatrix} + c_1 e^{\xi_1} \Xi_1 + c_2 e^{\xi_2} \Xi_2 \tag{36}$$

where Ξ_j is the eigenvector that corresponds to eigenvalue ξ_j. Thus

$$\begin{pmatrix} K_1 \\ \mu_1 \end{pmatrix} = \begin{pmatrix} \hat{K}_1^M \\ \hat{\mu}_1^M \end{pmatrix} + c_1 e^{\xi_1 t} \begin{pmatrix} \frac{1}{2}(\xi_1 - r - \delta) \\ 1 \end{pmatrix} + c_2 e^{\xi_2 t} \begin{pmatrix} \frac{1}{2}(\xi_2 - r - \delta) \\ 1 \end{pmatrix} \tag{37}$$

By using the constraints $K(0) = K_0$ and $\mu_1(T) = S_{K_1}$ we can calculate c_1 and c_2. We obtain

$$c_1 = \frac{2(K_{10} - \hat{K}_1^M) - (\xi_2 - r - \delta)(S_{K_1} - \hat{\mu}_1^M)e^{-\xi_2 T}}{(\xi_1 - r - \delta) - (\xi_2 - r - \delta)e^{(\xi_1 - \xi_2)T}} \tag{38}$$

$$c_2 = \frac{(\xi_1 - r - \delta)e^{-\xi_2 T}(S_{K_1} - \hat{\mu}_1^M)}{(\xi_1 - r - \delta) - (\xi_2 - r - \delta)e^{(\xi_1 - \xi_2)T}} - \frac{2e^{(\xi_1 - \xi_2)T}(K_{10} - \hat{K}_1^M)}{(\xi_1 - r - \delta) - (\xi_2 - r - \delta)e^{(\xi_1 - \xi_2)T}} \tag{39}$$

Using them together with (37) gives the solution for the finite horizon case ($T < \infty$), which we need for our model.

For case without anticipation we also need the solution for the infinite horizon. In this case the optimal behavior is determined by the stable manifold (i.e. $c_1 = 0$). Thus

$$K_1(t) = \hat{K}_1^M + e^{\xi_2 t}(K_{10} - \hat{K}_1^M) \tag{40}$$

$$\mu_1(t) = \hat{\mu}_1^M + e^{\xi_2 t}\frac{2(K_{10} - \hat{K}_1^M)}{\xi_2 - r - \delta} \tag{41}$$

$$I_1(t) = \delta\hat{K}_1^M + e^{\xi_2 t}\frac{2(K_{10} - \hat{K}_1^M)}{c(\xi_2 - r - \delta)} \tag{42}$$

Remark: Here c_1 is still not defined in closed form since the marginal salvage value $S_{K_1}(K_1(T))$ still depends on $K_1(T)$. This can only be done with the value function of the competition period which equal the salvage value. This also depends on the information structure of the competition period, i.e. open-loop or Markov perfect. Both cases will be studied in the corresponding section.

Appendix B. Open-loop solution of the competition period

Within this section we present the key parts of the analysis of the open-loop solution of the competition period, as already presented by Reynolds (1987). In the open-loop scenario optimal investments are considered as functions of time only, i.e. $I_i = I_i(t)$. The calculations are presented for firm 1.

The Hamiltonian of firm 1 reads

$$\mathcal{H}_1^O = (A - K_1 - K_2)K_1 - bI_1 - \frac{c}{2}I_1^2 + \mu_{11}(I_1 - \delta K_1) + \mu_{12}(I_2 - \delta K_2) \tag{43}$$

where μ_{ij} denotes firm *i*'s adjoint variable for the capital owned by firm *j*. The first order condition for investments

$$\mathcal{H}^O_{I_1} = -b - cI_1 + \mu_{11} = 0 \tag{44}$$

implies the following investment behavior

$$I_1 = \frac{\mu_{11} - b}{c}. \tag{45}$$

The adjoint equations for firm 1 read

$$\dot{\mu}_{11} = (r + \delta)\mu_{11} - (A - 2K_1 - K_2) \tag{46}$$

$$\dot{\mu}_{12} = (r + \delta)\mu_{12} + K_1 \tag{47}$$

Thus the behavior in the competition period is described by the 6-dimensional dynamical system (K_1, K_2, μ_{11}, μ_{12}, μ_{21}, μ_{22}). The unique equilibrium equals for $i = 1, 2$

$$\hat{K}^O = \frac{A - b(r+\delta)}{3 + \delta c(r+\delta)} \tag{48}$$

$$\hat{\mu}_{ii} = \frac{A c\delta + 3b}{3 + \delta c(r+\delta)} \tag{49}$$

$$\hat{\mu}_{ij} = -\frac{A - b(r+\delta)}{(r+\delta)(3 + \delta c(r+\delta))} \tag{50}$$

$$\hat{I}_i = \frac{\delta(A - b(r+\delta))}{3 + \delta c(r+\delta)} \tag{51}$$

To study the stability of the equilibrium we look at the eigenvalues of the Jacobian evaluated at the unique equilibrium, i.e.

$$\xi_1^O = \frac{r + \sqrt{r^2 + 4((r+\delta)\delta + \frac{1}{c})}}{2} \tag{52}$$

$$\xi_2^O = \frac{r - \sqrt{r^2 + 4((r+\delta)\delta + \frac{1}{c})}}{2} \tag{53}$$

$$\xi_3^O = \frac{r + \sqrt{r^2 + 4((r+\delta)\delta + \frac{3}{c})}}{2} \tag{54}$$

$$\xi_4^O = \frac{r - \sqrt{r^2 + 4((r+\delta)\delta + \frac{3}{c})}}{2} \tag{55}$$

$$\xi_5^O = r + \delta \tag{56}$$

$$\xi_6^O = r + \delta \tag{57}$$

Analogously to the monopolistic case it is obvious that $\xi_1^O, \xi_3^O, \xi_5^O, \xi_6^O > 0$ and $\xi_2^O, \xi_4^O < 0$, which again implies a saddle point.

The general solution of the dynamical system can be found from

$$\begin{pmatrix} K_1 \\ K_2 \\ \mu_{11} \\ \mu_{12} \\ \mu_{21} \\ \mu_{22} \end{pmatrix} = \begin{pmatrix} \hat{K}^O \\ \hat{K}^O \\ \hat{\mu}_{ii}^O \\ \hat{\mu}_{ij}^O \\ \hat{\mu}_{ij}^O \\ \hat{\mu}_{ii}^O \end{pmatrix} + c_1 e^{\xi_1^O t}\Xi_1^O + c_2 e^{\xi_2^O t}\Xi_2^O + c_3 e^{\xi_3^O t}\Xi_3^O$$
$$+ c_4 e^{\xi_4^O t}\Xi_4^O + c_5 e^{\xi_5^O t}\Xi_5^O + c_6 e^{\xi_6^O t}\Xi_6^O \tag{58}$$

where Ξ_i^O is the eigenvector that corresponds to eigenvalue ξ_i^O. Setting the constant corresponding to positive eigenvalues (i.e. c_1, c_3, c_5 and c_6) equal to zero we obtain the stable path. Thus we obtain

$$\begin{pmatrix} K_1 \\ K_2 \\ \mu_{11} \\ \mu_{12} \\ \mu_{21} \\ \mu_{22} \end{pmatrix} = \begin{pmatrix} \hat{K}^O \\ \hat{K}^O \\ \hat{\mu}_{ii}^O \\ \hat{\mu}_{ij}^O \\ \hat{\mu}_{ij}^O \\ \hat{\mu}_{ii}^O \end{pmatrix} + c_2 e^{\xi_2^O t}\begin{pmatrix} -\xi_2^O + r + \delta \\ \xi_2^O - r - \delta \\ -1 \\ -1 \\ 1 \\ 1 \end{pmatrix} + c_4 e^{\xi_4^O t}\begin{pmatrix} \xi_4^O - r - \delta \\ \xi_4^O - r - \delta \\ 3 \\ 1 \\ 1 \\ 3 \end{pmatrix} \tag{59}$$

Solving for the constants c_2 and c_4 by using $K_1(0) = K_{10}$ and $K_2(0) = K_{20}$ we get

$$c_2 = -\frac{(K_{10} - K_{20})}{2} c(\xi_2^O + \delta) \tag{60}$$

$$c_4 = \frac{(K_{10} + K_{20}) - 2\hat{K}_i^O}{2} \frac{c}{3}(\xi_4^O + \delta) \tag{61}$$

and

$$K_i(t) = \hat{K}^O + e^{\xi_2^O t}\frac{K_{i0} - K_{j0}}{2} + e^{\xi_4^O t}\left(\frac{K_{i0} + K_{j0}}{2} - \hat{K}_i\right) \tag{62}$$

$$I_i(t) = \delta\hat{K}^O + (\xi_2^O + \delta)e^{\xi_2^O t}\frac{K_{i0} - K_{j0}}{2} + (\xi_4^O + \delta)e^{\xi_4^O t}\left(\frac{K_{i0} + K_{j0}}{2} - \hat{K}_i\right) \tag{63}$$

$$\mu_{ii}(t) = \hat{\mu}_{ii} + c(\xi_2^O + \delta)e^{\xi_2^O t}\frac{K_{i0} - K_{j0}}{2} + c(\xi_4^O + \delta)e^{\xi_4^O t}\left(\frac{K_{i0} + K_{j0}}{2} - \hat{K}_i\right) \tag{64}$$

$$\mu_{ij}(t) = \hat{\mu}_{ij} + c(\xi_2^O + \delta)e^{\xi_2^O t}\frac{K_{i0} - K_{j0}}{2} + \frac{c}{3}(\xi_4^O + \delta)e^{\xi_4^O t}\left(\frac{K_{i0} + K_{j0}}{2} - \hat{K}_i\right) \tag{65}$$

By using (62) and (63) together with the objective function we are able to calculate the value function of both players explicitly. Due to the tedious calculation we only present the result for player 1 and $K_{20} = 0$, which we need as salvage function of the monopolist in the open-loop scenario. We get the following quadratic function

$$\begin{aligned} S(K_1) = {} & \underbrace{\left[\frac{Z_1}{r} + \hat{K}_i^O \frac{Z_2}{\xi_4^O - r} - (\hat{K}_i^O)^2 \frac{Z_5}{2\xi_4^O - r} - \frac{1}{2\xi_4^O - r}\frac{Z_6}{4}\right]}_{:=Y_1} \\ & + K_1 \underbrace{\left[-\frac{1}{\xi_4^O - r}\frac{Z_2}{2} - \frac{1}{\xi_2^O - r}\frac{Z_3}{2} + \frac{1}{\xi_2^O + \xi_4^O - r}\frac{Z_4}{2}\hat{K}_i^O + \frac{1}{2\xi_4^O - r}Z_5\hat{K}_i^O\right]}_{:=Y_2} \\ & + K_1^2 \underbrace{\left[-\frac{1}{\xi_2^O + \xi_4^O - r}\frac{Z_4}{4} - \frac{1}{2\xi_4^O - r}\frac{Z_5}{4}\right]}_{:=Y_3} \\ = {} & Y_1 + Y_2 K_1 + Y_3 K_1^2 \end{aligned} \tag{66}$$

with

$$\begin{aligned} Z_1 &= (A - \hat{K}^O)\hat{K}^O - b\delta\hat{K}^O - \frac{c}{2}\delta^2\hat{K}^O \\ Z_2 &= \big(-4 - c\delta(\xi_4^O + \delta)\big)\hat{K}^O + A - b(\xi_4^O + \delta) \\ Z_3 &= A - 2\hat{K}^O - (\xi_2^O + \delta)(b + c\delta\hat{K}^O) \\ Z_4 &= -2 - c(\xi_2^O + \delta)(\xi_4^O + \delta) \\ Z_5 &= -2 - \frac{c}{2}(\xi_4^O + \delta)^2 \\ Z_6 &= -\frac{c}{2}(\xi_2^O + \delta)^2 \end{aligned} \tag{67}$$

Appendix C. Value function parameters of Section 3

Calculated parameters of the value function of player 2 for a switch at $\tilde{T}$[8]

$$\begin{aligned} A_2 := {} & \int_0^\infty \frac{e^{-rt}}{2}\left(\left(e^{(\xi_2^O + \xi_4^O)t} - e^{2\xi_4^O t}\right) + c\left(\frac{\xi_2^O + \delta}{2}e^{\xi_2^O t} - \frac{\xi_4^O + \delta}{2}e^{\xi_4^O t}\right)^2\right) dt \\ = {} & -\frac{1}{2}\left[\frac{1}{\xi_2^O + \xi_4^O - r} - \frac{1}{2\xi_4^O - r} + c\left(\frac{\xi_2^O + \delta}{2}\right)^2 \frac{1}{2\xi_2^O - r}\right. \\ & \left. - c\frac{(\xi_2^O + \delta)(\xi_4^O + \delta)}{2}\frac{1}{\xi_2^O + \xi_4^O - r} + c\left(\frac{\xi_4^O + \delta}{2}\right)^2 \frac{1}{2\xi_4^O - r}\right] \end{aligned}$$

[8] Note that the lower integral bound equals $\tilde{T}$ and the discount factor $e^{-r(t-\tilde{T})}$. However, we used a transformation to obtain the above form, which is easier (i.e. the notation is easier) to deal with.

$$
\begin{aligned}
B_2 :=& \int_0^\infty e^{-rt}\Big[\frac{1}{2}(b + c(\delta\hat{K}_i^O - (\xi_4^O+\delta)e^{\xi_4^O t}\hat{K}_i^O))((\xi_2^O+\delta)e^{\xi_2^O t} \\
& - (\xi_4^O+\delta)e^{\xi_4^O t}) - \frac{1}{2}(A - 2\hat{K}_i^O(1-e^{\xi_4^O t}))(e^{\xi_2^O t} - e^{\xi_4^O t}) \\
& - e^{\xi_4^O t}(1-e^{\xi_4^O t})\hat{K}_i^O\Big]\ dt \\
=& -\frac{1}{2}\left(b + c\delta\hat{K}_1^O\right)\left(\frac{\xi_2^O+\delta}{\xi_2^O-r} - \frac{\xi_4^O+\delta}{\xi_4^O-r}\right) \\
& + \frac{c}{2}\hat{K}_1^O\left(\frac{(\xi_2^O+\delta)(\xi_4^O+\delta)}{\xi_2^O+\xi_4^O-r} - \frac{(\xi_4^O+\delta)^2}{2\xi_4^O-r}\right) \\
& + \frac{A}{2}\left(\frac{1}{\xi_2^O-r} - \frac{1}{\xi_4^O-r}\right) \\
& + \hat{K}_1^O\left(-\frac{1}{\xi_2^O-r} + \frac{1}{\xi_2^O+\xi_4^O-r} + \frac{2}{\xi_4^O-r} - \frac{2}{2\xi_4^O-r}\right) \\
C_2 :=& \int_0^\infty e^{-rt}\Big[(A - 2\hat{K}_i^O(1-e^{\xi_4^O t}))(1-e^{\xi_4^O t})\hat{K}_i^O \\
& - \frac{c}{2}(\hat{K}_i^O)^2(\delta - (\xi_4^O+\delta)e^{\xi_4^O t})^2 - b\hat{K}_i^O(\delta - (\xi_4^O+\delta)e^{\xi_4^O t})\Big]\ dt \\
=& A\hat{K}_i^O\left(\frac{1}{r} + \frac{1}{\xi_4^O-r}\right) - 2(\hat{K}_i^O)^2\left(\frac{1}{r} + \frac{2}{\xi_4^O-r} - \frac{1}{2\xi_4^O-r}\right) \\
& - \frac{c}{2}(\hat{K}_i^O)^2\left(\frac{\delta^2}{r} + \frac{2\delta(\xi_4^O+\delta)}{(\xi_4^O-r)} - \frac{(\xi_4^O+\delta)^2}{2\xi_4^O-r}\right) \\
& - b\hat{K}_i^O\left(\frac{\delta}{r} + \frac{\xi_4^O+\delta}{\xi_4^O-r}\right)
\end{aligned} \tag{68}
$$

Value function parameters for firm 1 if firm 1 enters the market.

$$
\begin{aligned}
A_1 :=& \int_0^\infty \frac{e^{-rt}}{2}\left((e^{(\xi_2^O+\xi_4^O)t} + e^{2\xi_4^O t}) - c\left(\frac{\xi_2^O+\delta}{2}e^{\xi_2^O t} + \frac{\xi_4^O+\delta}{2}e^{\xi_4^O t}\right)^2\right) dt \\
=& -\frac{1}{2}\Bigg[\frac{1}{\xi_2^O+\xi_4^O-r} + \frac{1}{2\xi_4^O-r} - c\Big(\frac{\xi_2^O+\delta}{2}\Big)^2\frac{1}{2\xi_2^O-r} \\
& - c\frac{(\xi_2^O+\delta)(\xi_4^O+\delta)}{2}\frac{1}{\xi_2^O+\xi_4^O-r} - c\left(\frac{\xi_4^O+\delta}{2}\right)^2\frac{1}{2\xi_4^O-r}\Bigg] \\
B_1 :=& \int_0^\infty e^{-rt}\Big[\frac{1}{2}(-b - c(\delta\hat{K}_i^O - (\xi_4^O+\delta)e^{\xi_4^O t}\hat{K}_i^O))((\xi_2^O+\delta)e^{\xi_2^O t} \\
& + (\xi_4^O+\delta)e^{\xi_4^O t}) - \frac{1}{2}(A - 2\hat{K}_i^O(1+e^{\xi_4^O t}))(e^{\xi_2^O t} + e^{\xi_4^O t}) \\
& - e^{\xi_4^O t}(1-e^{\xi_4^O t})\hat{K}_i^O\Big]\ dt \\
=& \frac{1}{2}\left(b + c\delta\hat{K}_1^O\right)\left(\frac{\xi_2^O+\delta}{\xi_2^O-r} + \frac{\xi_4^O+\delta}{\xi_4^O-r}\right) \\
& - \frac{c}{2}\hat{K}_1^O\left(\frac{(\xi_2^O+\delta)(\xi_4^O+\delta)}{\xi_2^O+\xi_4^O-r} + \frac{(\xi_4^O+\delta)^2}{2\xi_4^O-r}\right) \\
& + \frac{A}{2}\left(\frac{1}{\xi_2^O-r} + \frac{1}{\xi_4^O-r}\right) - \hat{K}_1^O\left(\frac{1}{\xi_2^O-r} + \frac{1}{\xi_2^O+\xi_4^O-r} + \frac{2}{2\xi_4^O-r}\right) \\
C_1 :=& \int_0^\infty e^{-rt}\Big[(A - 2\hat{K}_i^O(1+e^{\xi_4^O t}))(1-e^{\xi_4^O t})\hat{K}_i^O \\
& - \frac{c}{2}(\hat{K}_i^O)^2(\delta - (\xi_4^O+\delta)e^{\xi_4^O t})^2 - b\hat{K}_i^O(\delta - (\xi_4^O+\delta)e^{\xi_4^O t})\Big]\ dt \\
=& A\hat{K}_i^O\left(\frac{1}{r} + \frac{1}{\xi_4^O-r}\right) - 2(\hat{K}_i^O)^2\left(\frac{1}{r} - \frac{1}{2\xi_4^O-r}\right) \\
& - \frac{c}{2}(\hat{K}_i^O)^2\left(\frac{\delta^2}{r} + \frac{2\delta(\xi_4^O+\delta)}{(\xi_4^O-r)} - \frac{(\xi_4^O+\delta)^2}{2\xi_4^O-r}\right) \\
& - b\hat{K}_i^O\left(\frac{\delta}{r} + \frac{\xi_4^O+\delta}{\xi_4^O-r}\right)
\end{aligned} \tag{69}
$$

Parameters for the general solution of the optimal control problem of the monopolist, if a certain capital stock $\bar{K}$ should be reached at T.

$$
\begin{aligned}
\bar{c}_2(\bar{K}) &= \frac{\bar{K} - K_{10}e^{\xi_1^M T} - \hat{K}_1^M(1-e^{\xi_1^M T})}{\frac{1}{2}(e^{\xi_2^M T} - e^{\xi_1^M T})(\xi_2^M - r - \delta)} \\
\bar{c}_1(\bar{K}) &= \frac{K_{10} - \hat{K}_1^M - \frac{\bar{c}_2}{2}(\xi_2^M - r - \delta)}{\frac{1}{2}(\xi_1^M - r - \delta)}
\end{aligned} \tag{70}
$$

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